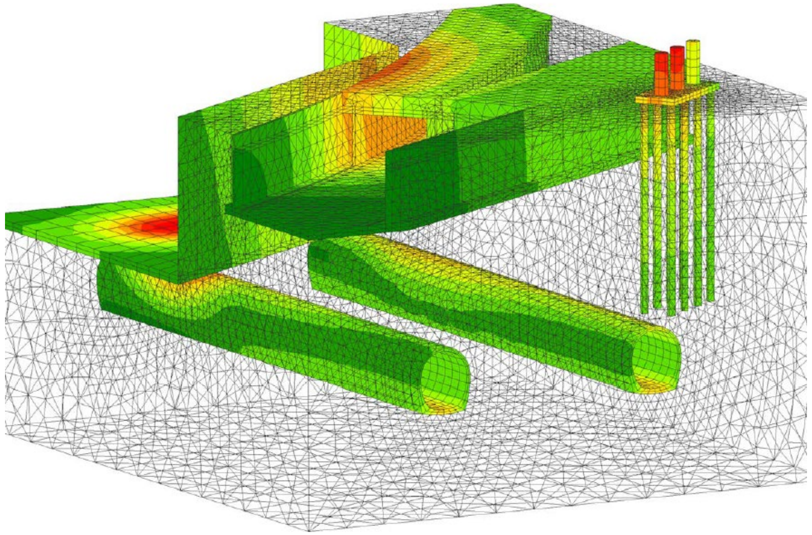

CFMS

Geomechanics

LECTURE 10

ON NUMERICAL MODELLING IN GEOTECHNICS

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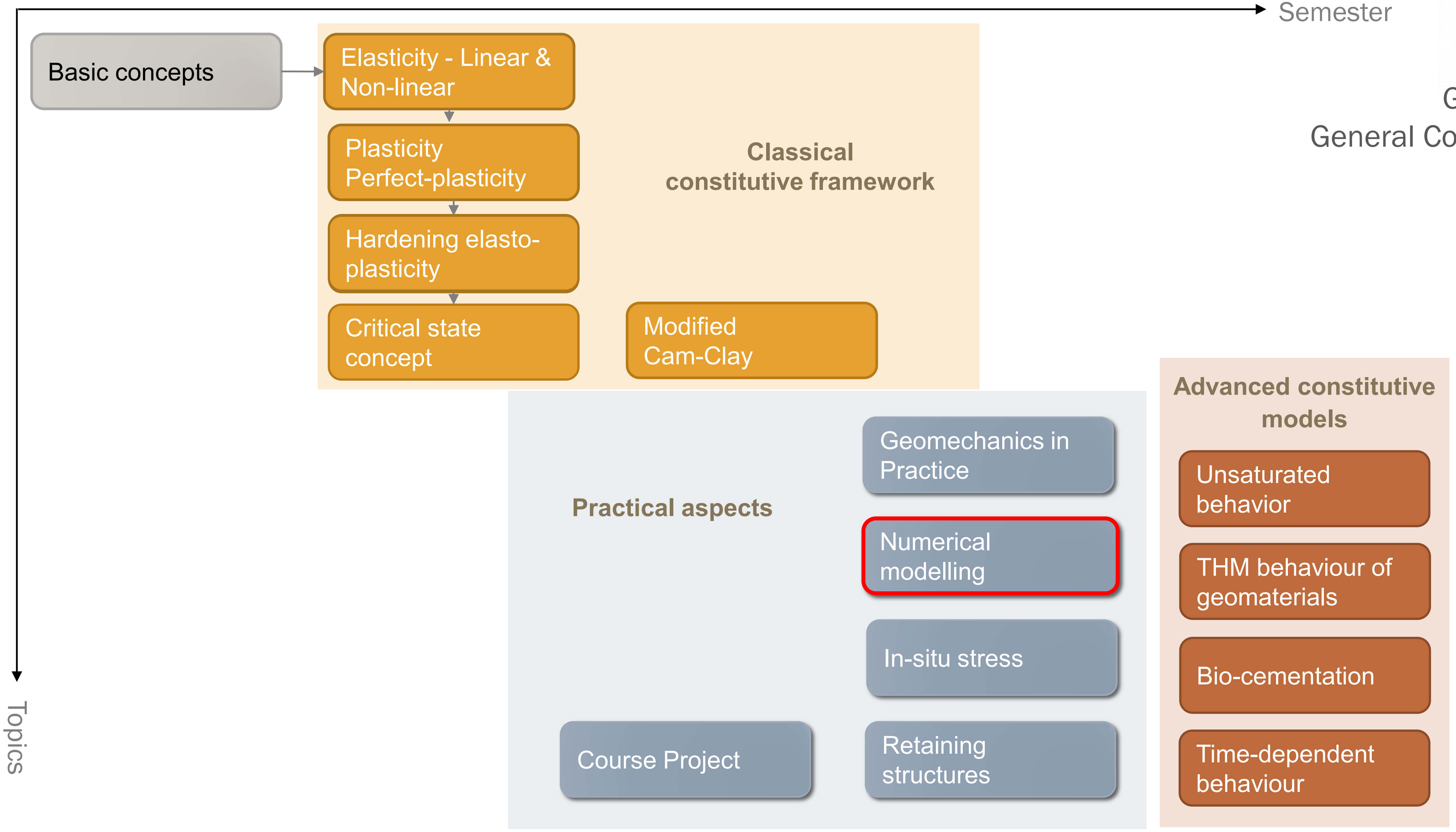
Laboratory of soil mechanics – Fall 2024

18.11.2024

Access the QUIZ

https://etc.ch/R4mA

Geomechanics
General Course overview



Content

- Introduction
- Model geometry
- Constitutive models
- Soil-structure interaction
- Initial and boundary conditions
- Construction phases
- Environmental couplings
- Solving methods
- Conclusion

Introduction

GOALS OF NUMERICAL MODELS

KEY ELEMENTS

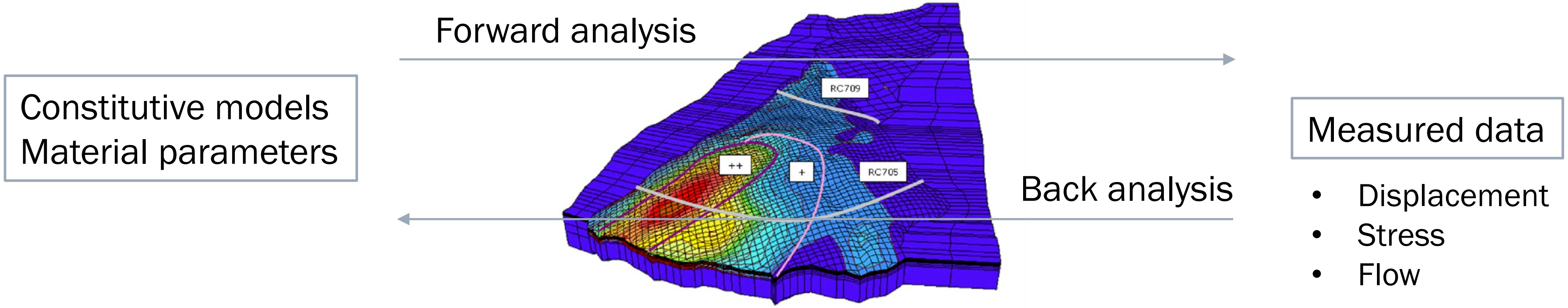
Goals of numerical models

Forward analysis

- Verify geotechnical work design with respect to limit states in design codes
- Test different design variants before construction phase starts

Back analysis

- Provide a theoretical explanation for observed behavior
- Improve the numerical model (constitutive laws, parameters)



La Frasse Landslide
Commend et al. (2004)

Key elements in the construction of a numerical model

- Selection of model elements and geometry (types of soil, structural elements, 2D-3D, …)
- Choice of suitable constitutive models
- Modelling of structural interactions
- Definition of the initial state and boundary conditions
- Definition of the project phases
- Consideration of the effect of environmental factors and how to couple them with the mechanical behavior

Model geometry

2D VS 3D MODELS

Model geometry

The model geometry should account for all phenomena that affect the behavior of the studied problem, including the surrounding structures

3D can often lead to more complex models than necessary

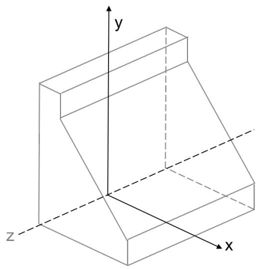
- If the geometry is complex and requires 3D model, other aspects of the analysis can be simplified

2D simplifications come from the consideration of symmetric properties or invariance of the problem

- Most of geotechnical problems are assumed in plane strain or axisymmetric conditions
- Often, 2D analyses are performed on multiple sections to avoid 3D model

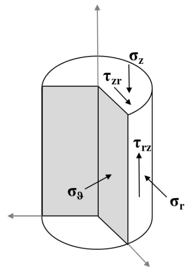
Simplified 2D analysis

- **Plane strain** conditions
  - Strip foundations
  - Deep tunnels
  - Retaining structures (walls, diaphragms)



$$[\varepsilon] = \begin{bmatrix} \varepsilon_x & \gamma_{xy} & 0 \\ \gamma_{yx} & \varepsilon_y & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- **Axi-symmetric** conditions
  - Single Pile
  - Circular flat footings
  - Boreholes and vertical shaft excavations



$$[\varepsilon] = \begin{bmatrix} \varepsilon_r & 0 & \gamma_{rz} \\ 0 & \varepsilon_\vartheta & \gamma_{\vartheta z} \\ \gamma_{zr} & \gamma_{z\vartheta} & \varepsilon_z \end{bmatrix}$$

- **Plane stress** conditions (rarely applied in geomechanics)

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Constitutive models

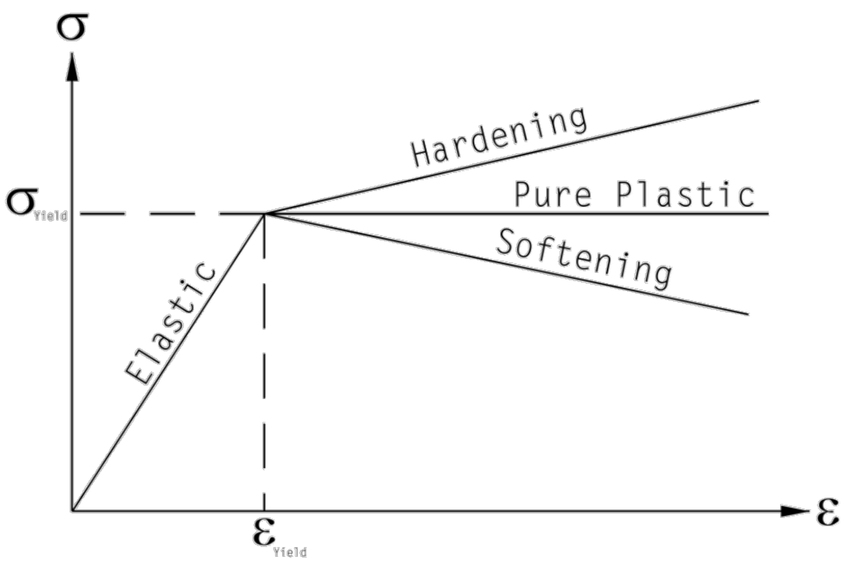
CHOICE OF THE CONSTITUTIVE MODEL

MODEL PARAMETERS

Choice of the constitutive model

Dependent on:

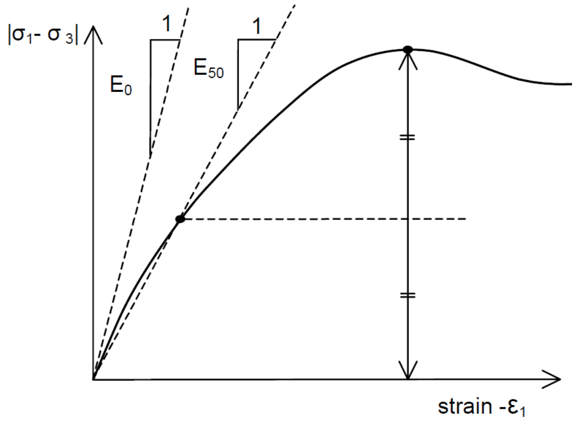
- Material behavior
- Type and complexity of the problem
- Scale of the problem
- Expected range of deformations
- Accuracy requirements

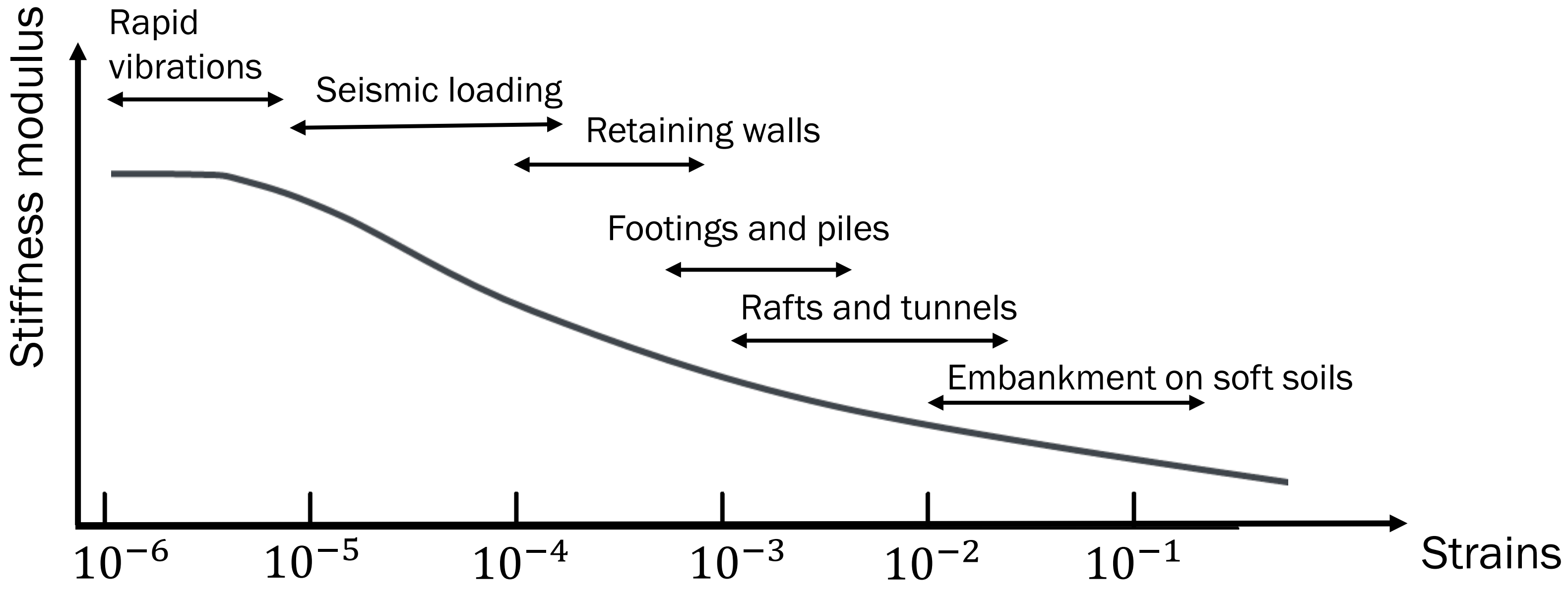


For certain works, due to the applied safety factors, elastic models can be sufficient

Determination of model parameters

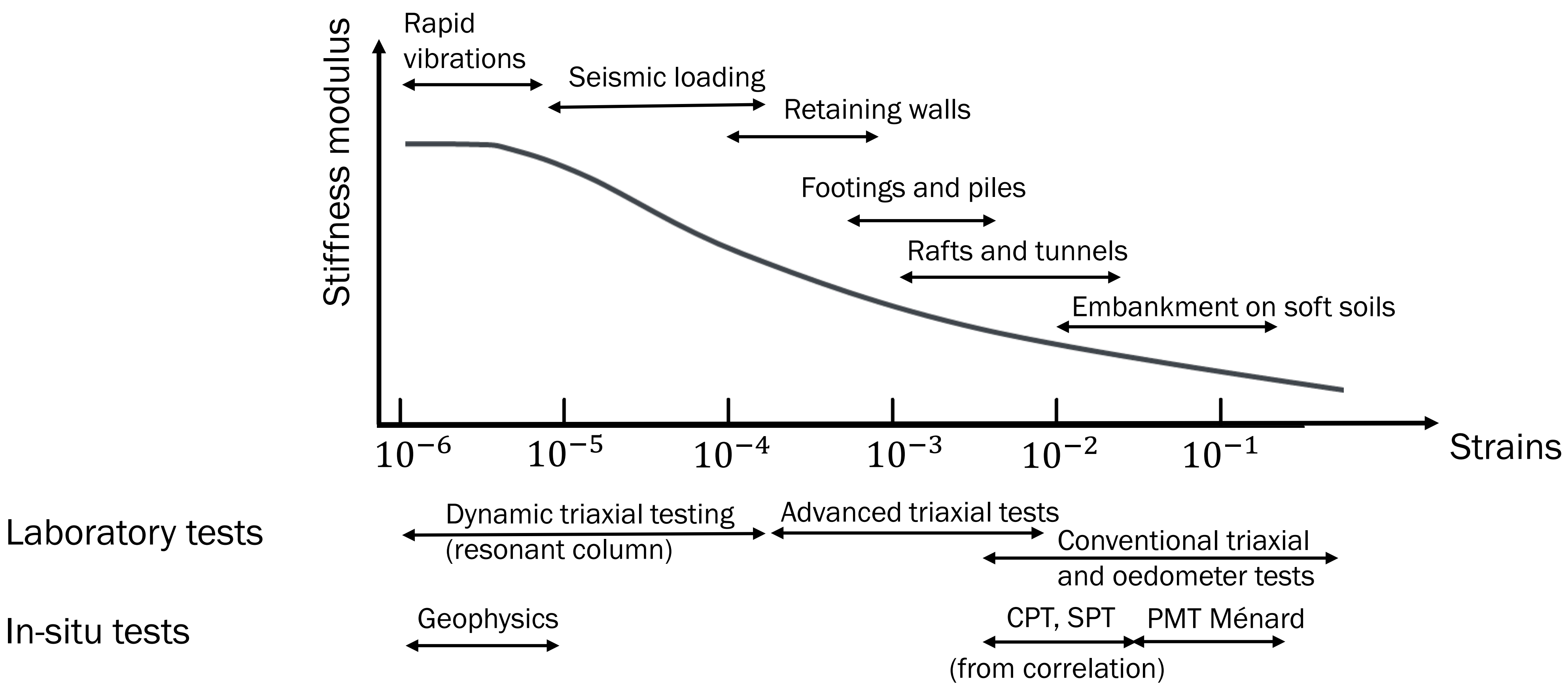
Often in practice, the non-linearity is approximated by a linear elastic behaviour
Therefore the "apparent" stiffness parameters (E) depends on the range of expected deformation (e.g., secant or tangent modulus)





Determination of model parameters

Laboratory and in-situ tests



Soil-structure interaction

STRUCTURAL ELEMENT

SOIL-STRUCTURE COUPLING

Choice of the structural element type

- Beam elements
  - For structures that can be loaded in tension/compression, shear and moment
    - Navier-Bernoulli: neglect shear strain (thin elements)
    - Timoshenko: consider shear strain (thick elements)
  - Pile, retaining structure in 2D plane strain

- Plate or shell elements
  - For structures loaded that are susceptible to arching effects
  - As for beam elements, shear strain can be neglected for thin elements
  - Retaining structure, raft

- Membranes
  - For structures that can only be loaded under tension
  - Geomembranes

Soil-structure coupling

General principle

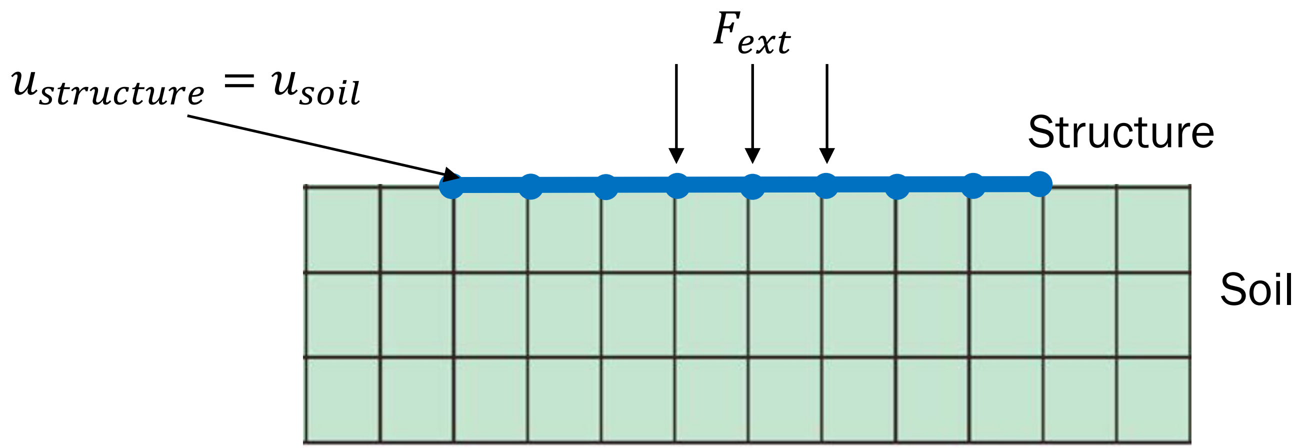
$$\boldsymbol{K}^{e} \cdot \boldsymbol{u}^{e} = \boldsymbol{F}^{ext} - \boldsymbol{R}_{s}$$

K^e	Stiffness matrix of the structural element (e.g., EI)
u^e	Degree of freedoms of the nodes
F^{ext}	External load
R_s	Interaction forces from the soil's reaction

Soil-structure coupling

Direct coupling

- Shared mesh nodes that experience the same deformations



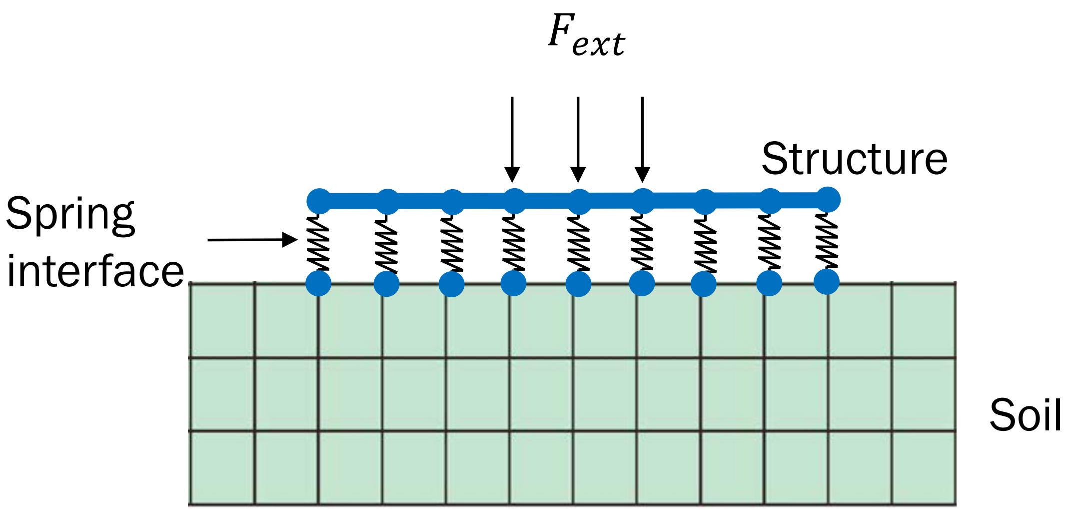
Easy and direct implementation

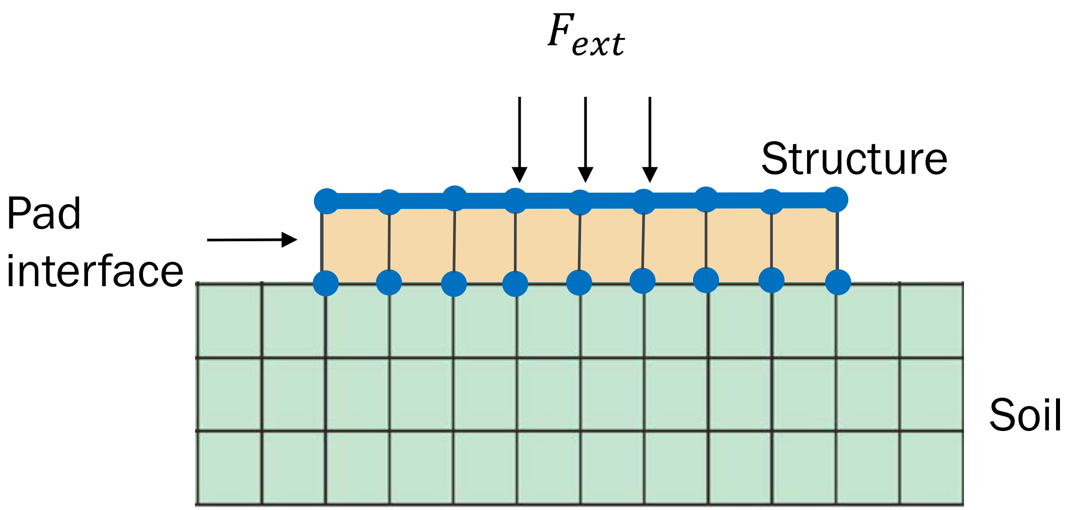
Limitation for non-linear effects between the structure and the soil
→ Implementation of an interface

Soil-structure coupling

Using an interface element

- Spring elements or pad elements are introduced to control the contact interface and simulate its nonlinear nature





Initial and boundary conditions

BOUNDARY OF THE MODEL

INITIAL STRESS CONDITIONS

Boundaries of the model

Typical boundary conditions of the model

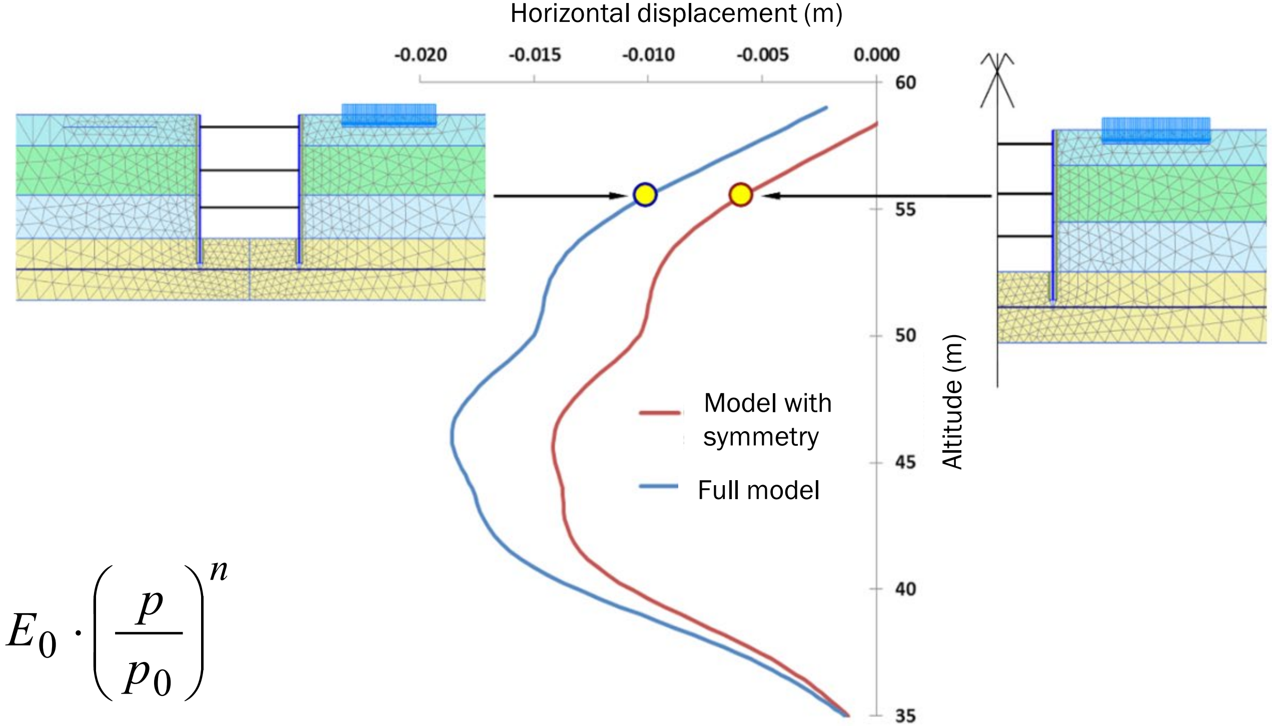
- Lateral : rollers, similar to symmetry boundary conditions
  - No normal deformation
- Bottom : fixed constraint
  - Displacement are zero in all direction

When you use the symmetry conditions, make sure this assumption is valid!

Depth of the model

- Insufficient depth can lead to underestimation of the settlement
- Significant depth can lead to overestimation of the settlement
  - Constant soil stiffness with depth may lead to unrealistic models
  - Non-linear evolution of the stiffness can improve the model

$$E_i = E_0 \cdot \left(\frac{p}{p_0}\right)^n$$



Initial stress conditions

Initialization of the stress state is a very important step in numerical modelling

Goal: define a stress field according to various factors (dead weight of the soil, pore pressure, loading history, etc…)

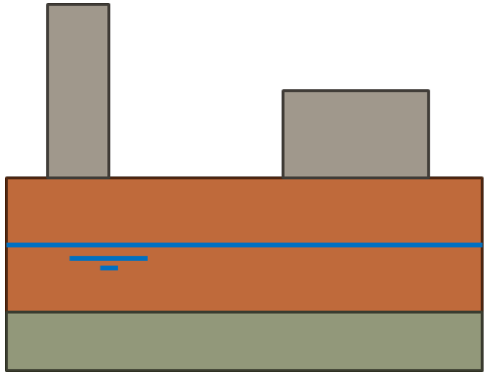
K0 method $$\sigma'_{h0} = K_0\, \sigma'_{v0}$$

σ'_{h0} Initial horizontal effective stress

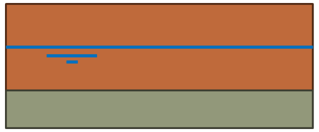
σ'_{v0} Initial vertical effective stress

K_0 Coefficient of lateral stress at rest (more details on lecture 12)

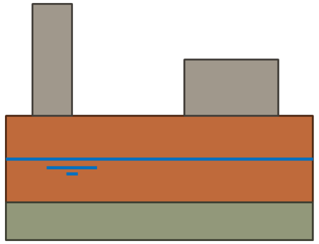
In case of existing buildings (or similar)



1. K0 method (without any building)



2. Initial calculation step with the existing buildings



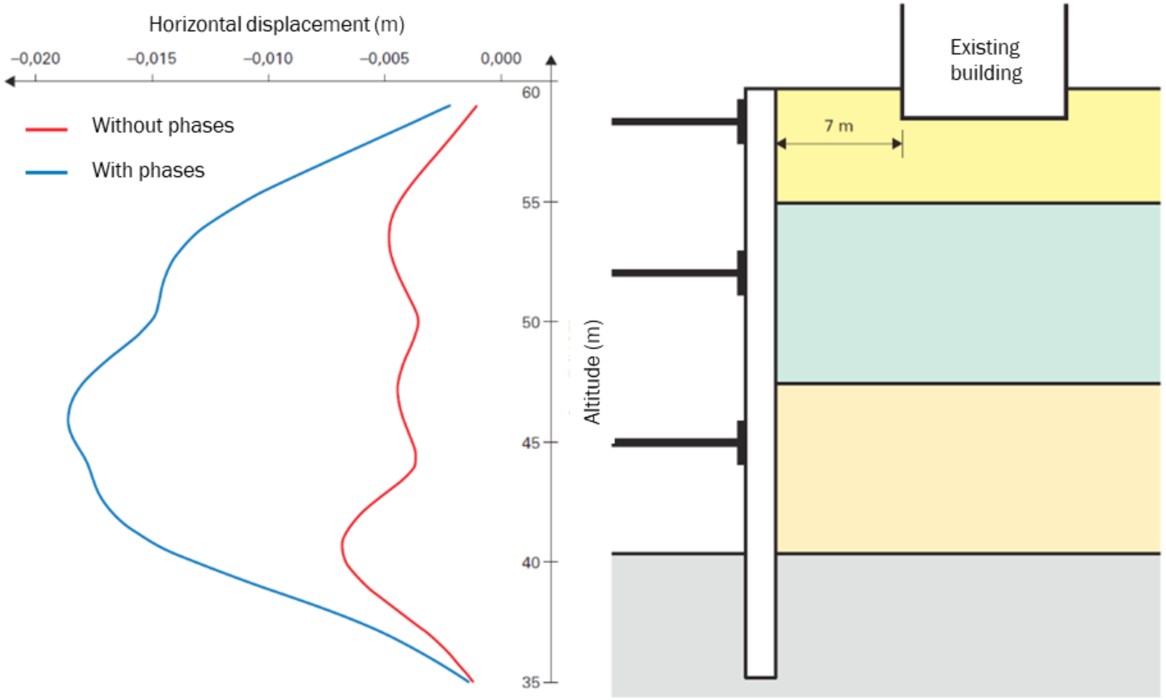
Construction phases

Construction phases

One of the main advantage of numerical modelling is the ability to model the loading history (prior and during the construction stage)

Omitting these aspects can lead to underestimation of the displacements and on the actions on the structural elements

An optimum and safe design does not only consider the final stage, but all the construction phases



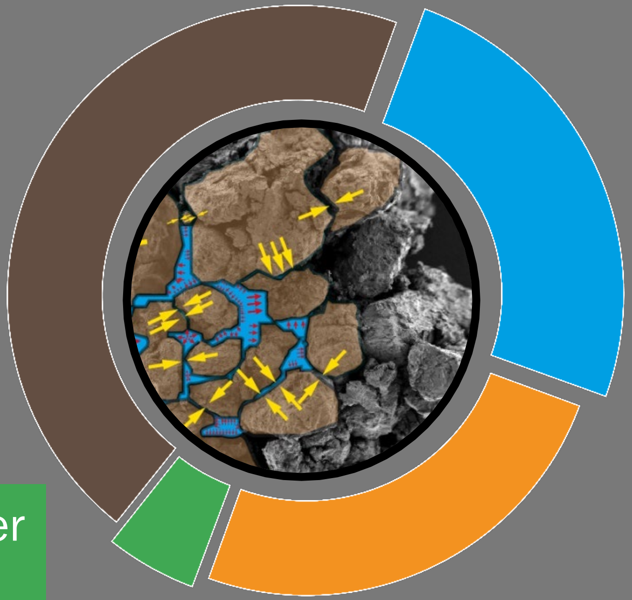
Environmental couplings

ENVIRONMENTAL COUPLINGS

HYDRO-MECHANICAL COUPLINGS

Environmental coupling

Multiphases



solid

water

gas

Organic matter environment

between

Solid mechanics

Fluid mechanics

Thermo-mechanics

Chemo-mechanics

Bio-mechanics

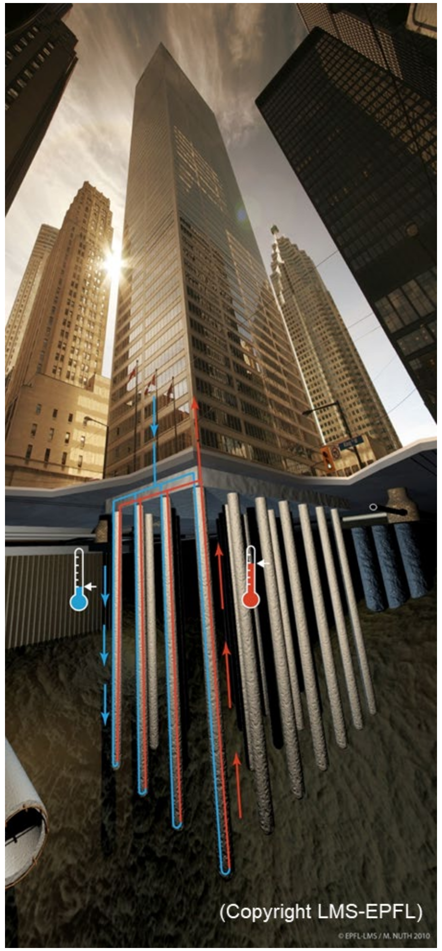
Multiphasic porous media

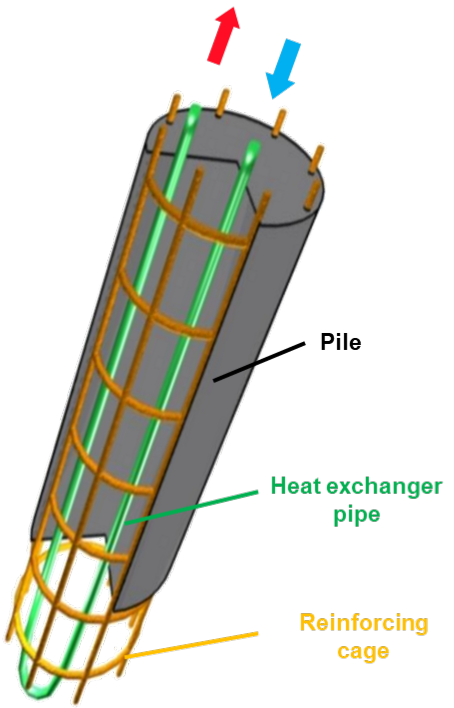
↕

Multi-physical processes

Environmental couplings

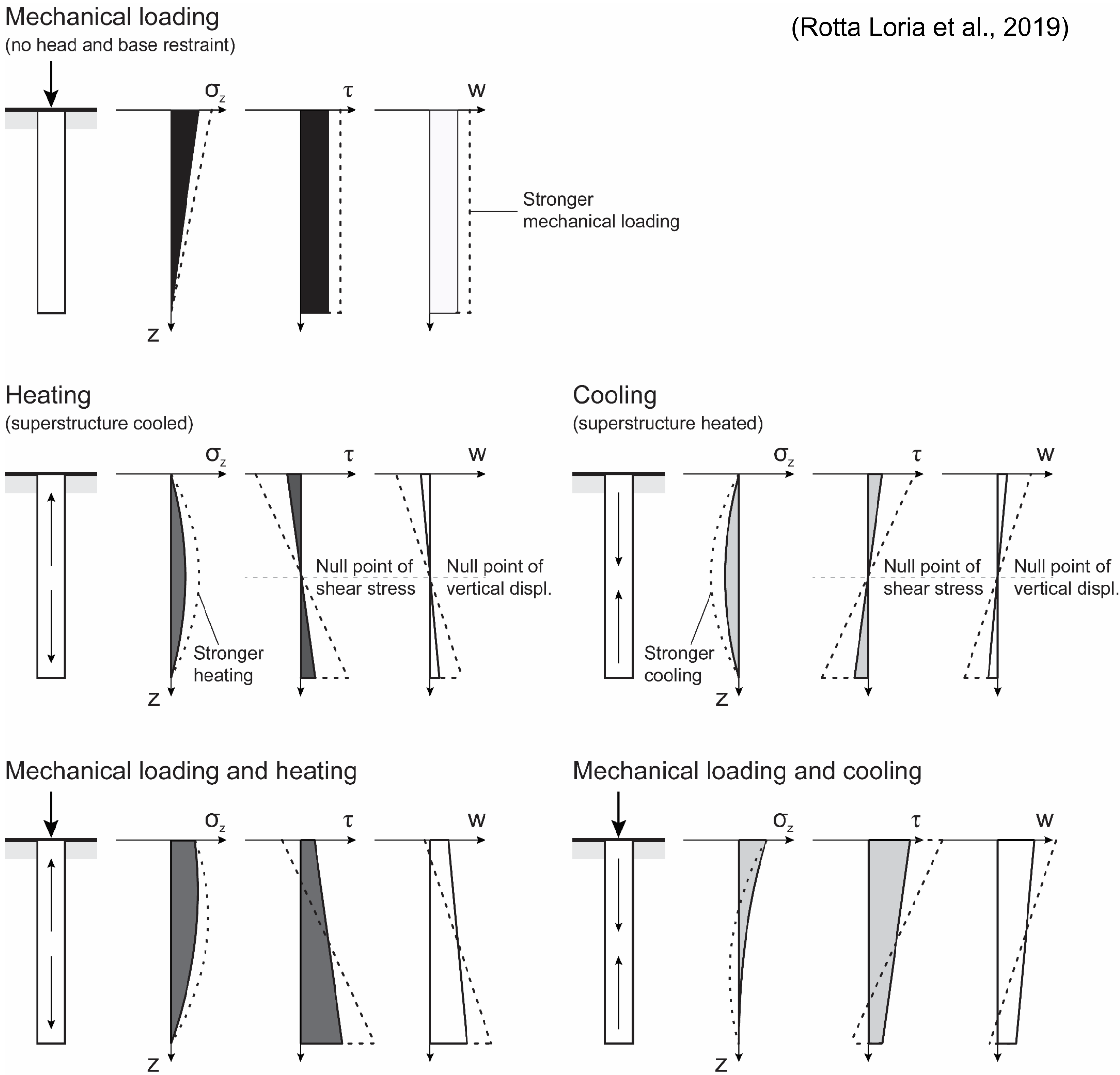
Energy Geostructures





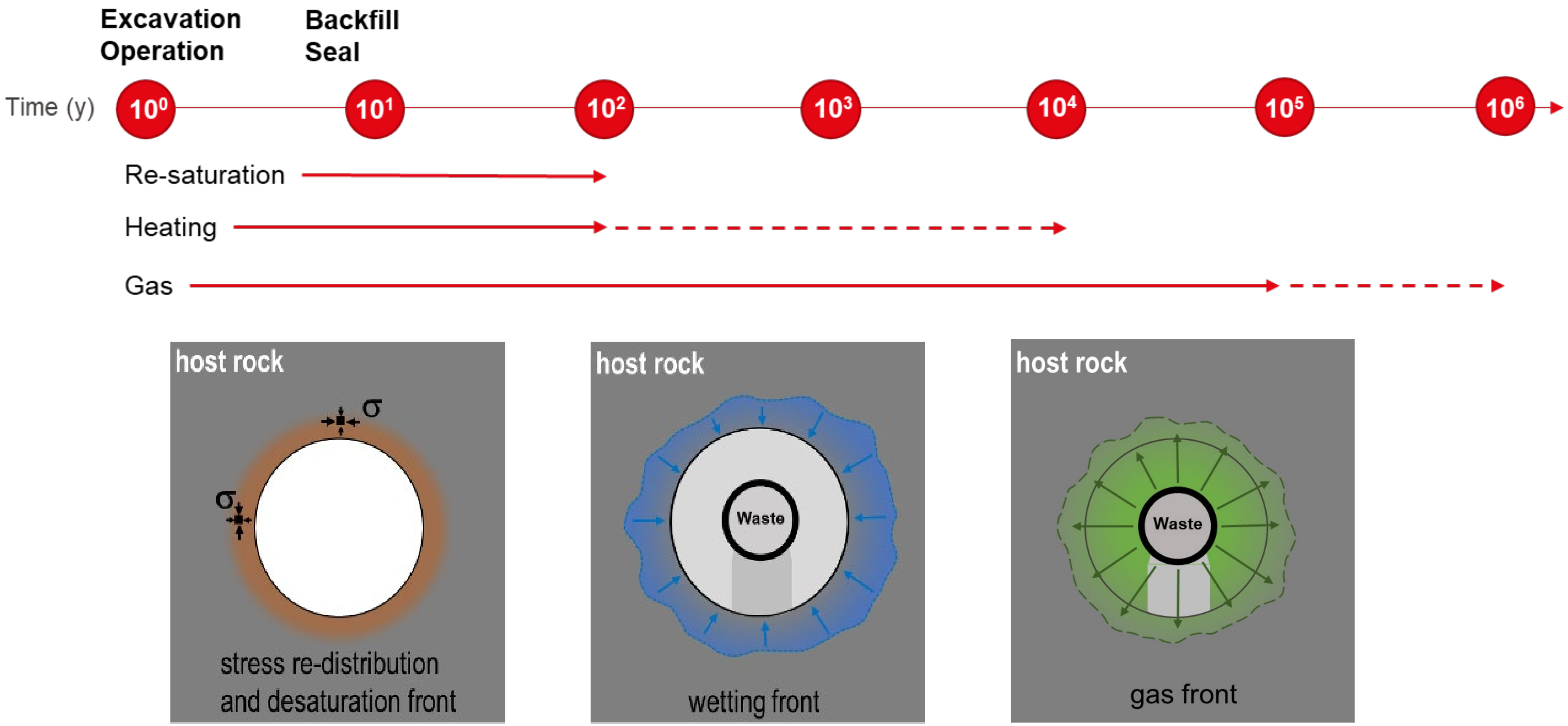
(Copyright LMS-EPFL)

(Rotta Loria et al., 2019)



Environmental couplings

Nuclear Waste Disposal

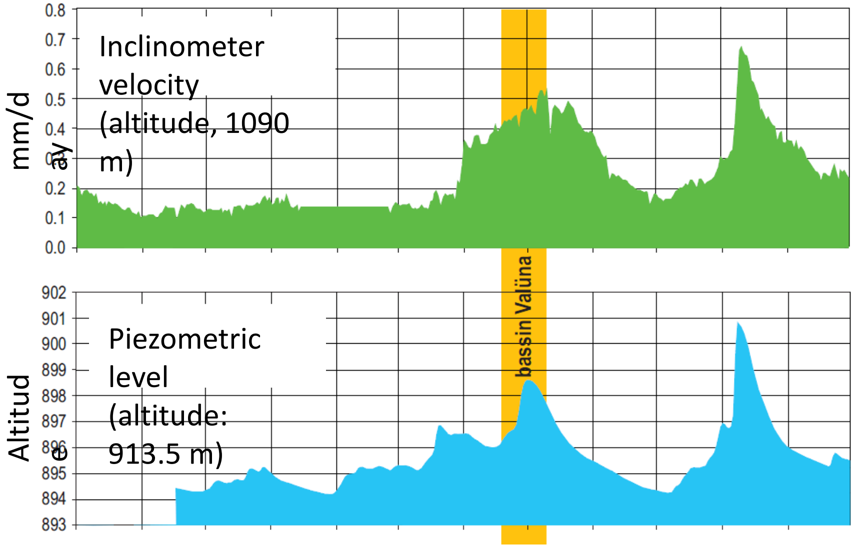


Hydro-mechanical coupling

Landslide example

H → M

Hydraulics on Mechanics

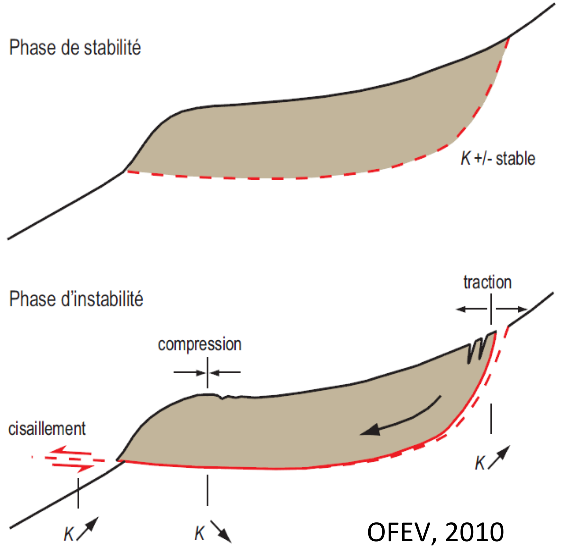


OFEV, 2010

High piezometric levels (i.e. **high pressures**) cause displacements and/or increase of slide velocity

M → H

Mechanics on Hydraulics



OFEV, 2010

Movements (i.e. **deformations**) induce change in permeability

Hydro-mechanical coupling

H → M

Influence of pore pressure on mechanical equilibrium in saturated soils:

Terzaghi's effective stress:

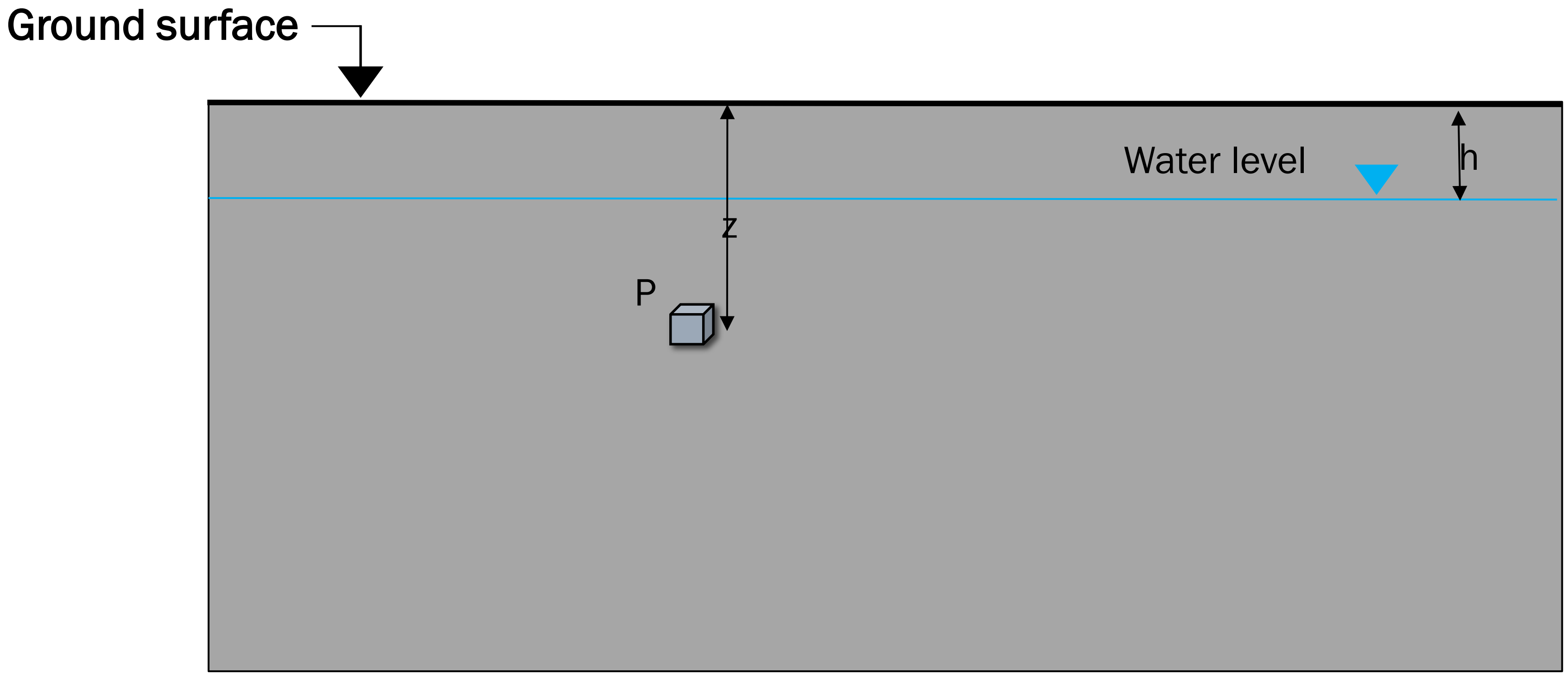
$$d\sigma'_{ij} = d\sigma_{ij} - dp_w\delta_{ij}$$

New constitutive law that relates the **effective stress tensor** to the strain rate tensor by the constitutive relationship:

$$d\sigma'_{ij} = D_{ijkl}d\varepsilon_{kl}$$

→ Pore pressure changes modify the effective stress which in turn causes **deformations**

Hydro-mechanical coupling

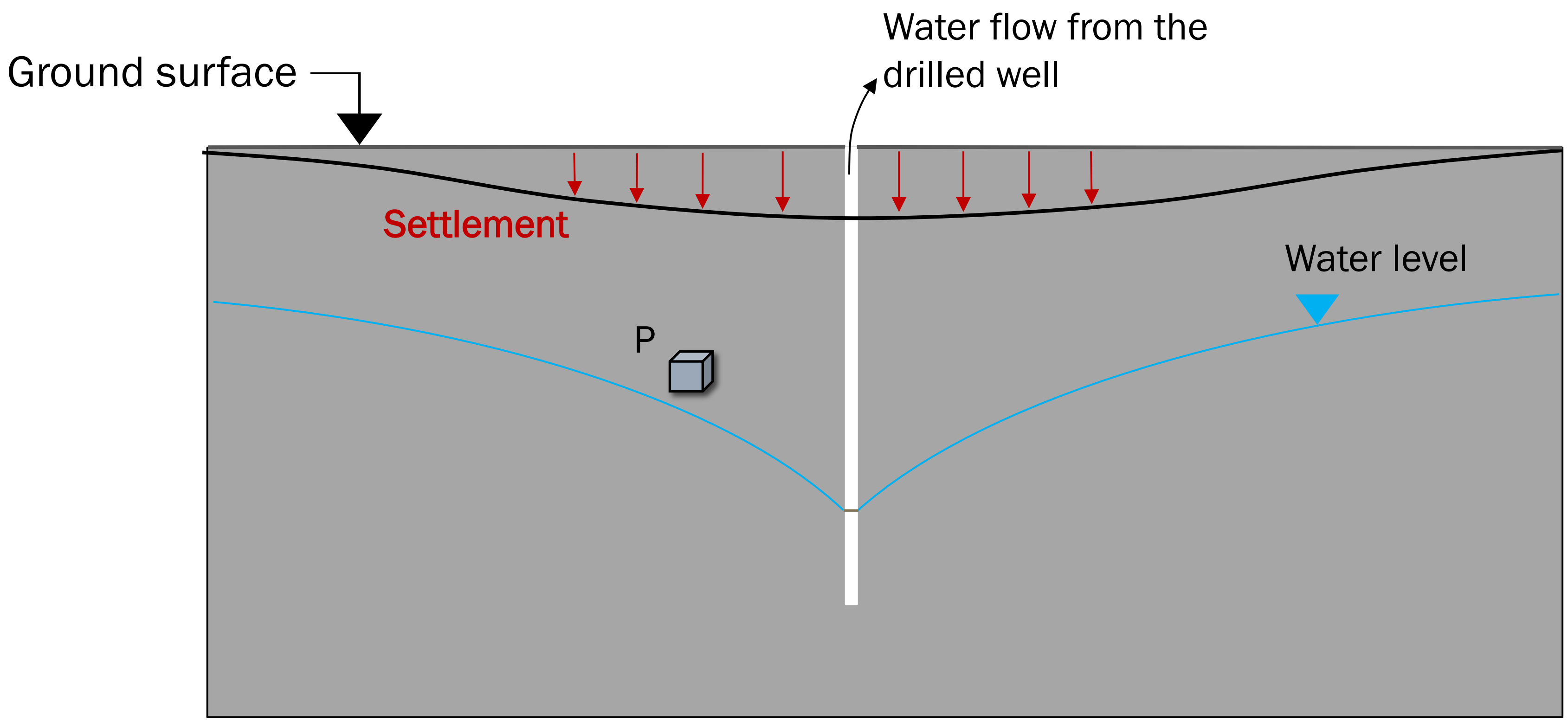


Stress state at point P:

$$\left.\begin{aligned} \sigma_v &= \gamma_{sat} z \\ p_w &= \gamma_w (z - h) \end{aligned}\right\} \sigma'_v = \sigma_v - p_w$$

$$\sigma'_h = k_0 \sigma'_v$$

Hydro-mechanical coupling



Pore water pressure decrease at point P causes an increase of effective stress

$$d\sigma'_{ij} = d\sigma_{ij} - dp_w\delta_{ij}$$

Deformation (i.e. settlements) are induced

$$d\varepsilon_{kl} = C_{ijkl}d\sigma'_{ij}$$

Hydro-mechanical coupling

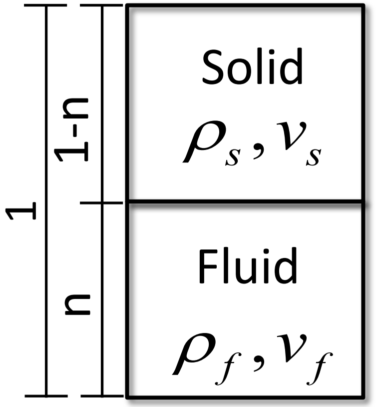
The two equations expressing the **Hydro-mechanical interactions**:

1) Balance of momentum: $div(\sigma) + \rho g = 0$

Consider Terzaghi's effective stress → $\boxed{div(\sigma') + \nabla p_w + \rho g = 0}$

2) Balance of mass (mass conservation): $\partial_t \rho + div(\rho v) = 0$

Write the mass balance equation for both solid and fluid parts and add them:



Solids: $\partial_t (1-n)\rho_s + div((1-n)\rho_s v_s) = 0$

$$-\rho_s \partial_t n + (1-n)\partial_t \rho_s + (1-n)\rho_s div(v_s) + (1-n) v_s \cancelto{0}{\nabla \rho_s} = 0$$

$$-\partial_t n + (1-n)\frac{\partial_t \rho_s}{\rho_s} + div(v_s) - n\,div(v_s) = 0 \qquad (1)$$

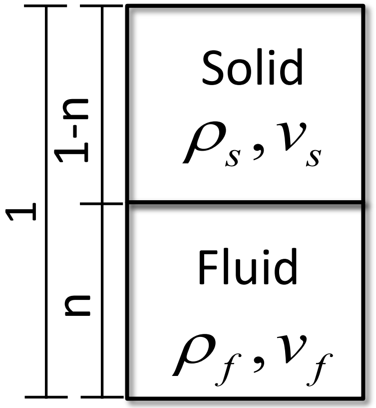
Fluids: $\partial_t n \rho_f + div(n \rho_f v_f) = 0$

$$\rho_f \partial_t n + n \partial_t \rho_f + n \rho_f div(v_f) + n v_f \cancelto{0}{\nabla \rho_f} = 0$$

$$\partial_t n + n \frac{\partial_t \rho_f}{\rho_f} + n\,div(v_f) = 0 \qquad (2)$$

Hydro-mechanical coupling

Sum of the two equations: $(1-n)\dfrac{\partial_t \rho_s}{\rho_s} + div(v_s) + n\dfrac{\partial_t \rho_f}{\rho_f} + n div(v_f - v_s) = 0$ (1)+(2)



Considering the **Darcy's law**:

$$v_{rf} = n\left(v_f - v_s\right) = -K\nabla(p_w + \rho_f g x)$$

It results:

$$(1-n)\frac{\partial_t \rho_s}{\rho_s} + n\frac{\partial_t \rho_f}{\rho_f} + div(v_s) - div K\nabla(p_w + \rho_f g x) = 0$$

Considering **compressibility β**:

$$\beta = \frac{\partial_{p_w} \rho}{\rho}$$

$$\partial_t \rho(p_w) = \partial_{p_w} \rho \partial_t p_w$$

$$\frac{\partial_t \rho_s}{\rho_s} = \beta_s \partial_t p_w$$

$$\frac{\partial_t \rho_f}{\rho_f} = \beta_f \partial_t p_w$$

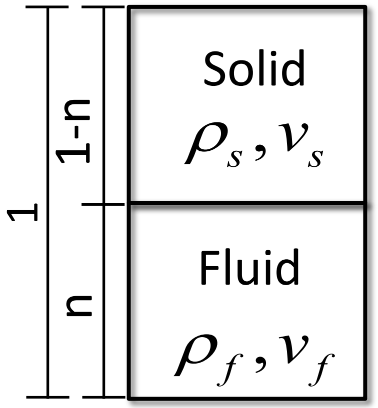
Hydro-mechanical coupling

$$\left[n\beta_f + (1-n)\boxed{\beta_s}\right]\partial_t p_w - divK\nabla(p_w + \rho_f gx) + \boxed{div(v_s)} = 0$$

β_s = 0 for an isotropic incompressible solid

$$div\left(v_s\right) = \partial_t \varepsilon_v$$

Volumetric strain rate



$$\rightarrow \boxed{\underbrace{\boxed{n\beta_f \partial_t p_w}}_{\text{Storage}} - \underbrace{\boxed{divK\nabla(p_w + \rho_f gx)}}_{\text{Flow}} + \underbrace{\boxed{\partial_t \varepsilon_v}}_{\text{Volume change}} = 0}$$

Field equations expressing the hydro-mechanical interaction:

1) Balance of momentum $div(\sigma') + \nabla p_w + \rho g = 0$

2) Mass conservation $n\beta_f \partial_t p_w - divK\nabla(p_w + \rho_f gx) + \partial_t \varepsilon_v = 0$

Hydro-mechanical coupling

Different types of coupling

Strong coupling / two-ways coupling: M ←→ H

Examples:

a) For the resolution of the consolidation equation (settlement of a foundation)
b) Needed when large variations of effective stress are expected

Weak coupling / one-way coupling: H → M

Examples: sufficient when small variations of effective stress are expected

More HM couplings if we consider partial saturation. For example, the increase in suction increases the strength and the stiffness of a soil H → M and at the same time causes a reduction of volume affecting therefore the permeability M → H.

Solution methods

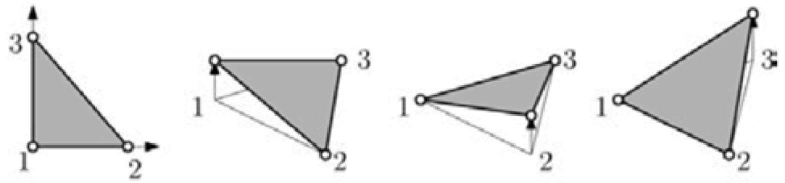
MODEL ELEMENTS

SOLUTION METHODS

Model elements

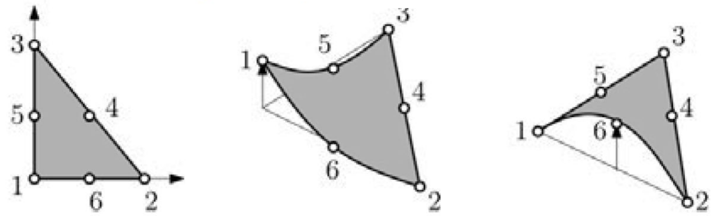
Most of finite element numerical models use elements with linear or quadratic interpolation functions

Linear elements
(lower order)



- Linear shape function (constant stress within a single element)
- Computationally less expensive
- Need a large number of elements to solve high stress gradients

Quadratic elements
(higher order)



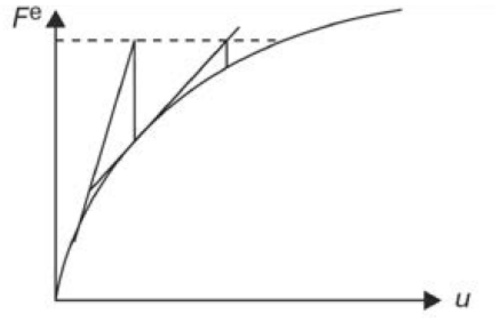
- Quadratic shape function (linear stress variation within a single element
- Computationally more expensive
- Allows a better approximation of the solution

In general, quadratic elements are recommended for accurate stress distribution

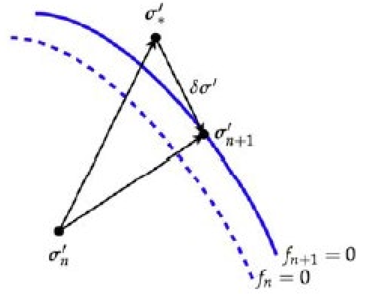
Solution methods

Global solution methods

- Use of algorithms (ex. Newton-Raphson) to solve the force equilibrium equations



Newton-Raphson method



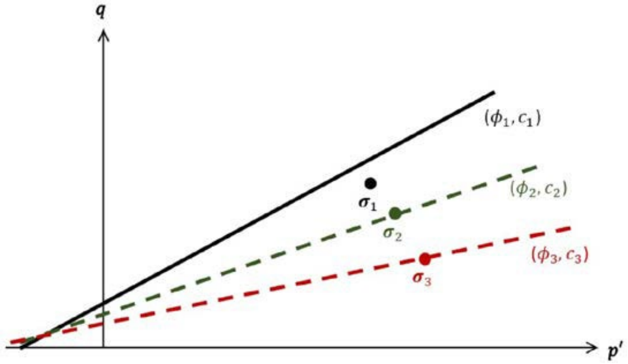
Radial return method

Local solution methods

- For each element, the stress state is corrected to satisfy the consistency condition ($f(\boldsymbol{\sigma}) \leq 0$); popular methods include the radial return method (Simo and Hughes, 1998)

C-Phi reduction

- The shear parameters of the soil (c', ϕ') are gradually reduced until parameters leading to failure are found for a given stress state. The reduction factor can be considered as partial safety factor for the design



C-Phi reduction

Conclusion

Conclusion

The objectives must be defined prior to any step

Although many situations can be simulated, the best choice is often not the most complex model

An excess of complexity can lead to numerical models that are difficult to control, especially from the large number of required parameters

A recommended approach is to start from simplified model (e.g., elasticity), and to progressively increase the complexity. This would allow a better control and comparison of the results

Thank you for your attention